

Schitz 8.8(a)

$$R^{\alpha}_{\beta\mu\nu} = \frac{1}{2} g^{\alpha\sigma} (g_{\sigma\nu,\beta\mu} - g_{\sigma\mu,\beta\nu} + g_{\beta\mu,\sigma\nu} - g_{\beta\nu,\sigma\mu})$$

weak field limit: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\Rightarrow R^{\alpha}_{\beta\mu\nu} = \frac{1}{2} (\eta^{\alpha\sigma} + h^{\alpha\sigma}) (h_{\sigma\nu,\beta\mu} - h_{\sigma\mu,\beta\nu} + h_{\beta\mu,\sigma\nu} - h_{\beta\nu,\sigma\mu})$$

$$= \frac{1}{2} \eta^{\alpha\sigma} (h_{\sigma\nu,\beta\mu} - h_{\sigma\mu,\beta\nu} + h_{\beta\mu,\sigma\nu} - h_{\beta\nu,\sigma\mu}) + \mathcal{O}(h^2)$$

$$= \boxed{\eta^{\alpha\sigma} R_{\sigma\beta\mu\nu} + \mathcal{O}(h^2_{\alpha\mu})}$$